NASA TECHNICAL TRANSLATION NASA TT F - 12,803

NASA TT F - 12,803

VISCOUS LIQUID FLOW IN THE INITIAL SEGMENT OF A FLAT CHANNEL WITH POROUS WALLS

V. N. Varapayev

Translation of "Techeniye Vyazkoy Zhidkosti V Nachal'nom Uchastke Ploskogo Kanala S Poristymi Stenkami," Izvestiya Akademii Nauk SSSR, Mekhanika Zhidkosti i Gaza, No. 4, pp. 178-181, July-Aug. 1969.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 FEBRUARY 1970

NASA TT F - 12,803

VISCOUS LIQUID FLOW IN THE INITIAL SEGMENT OF A FLAT CHANNEL WITH POROUS WALLS

V. N. Varapayev

ABSTRACT. The purpose of this article is to define the nature of the flow in the initial segment of a flat channel with a permeable and impermeable bottom. The investigation is conducted by means of a numerical solution of the Navier-Stokes equations by the finite-difference method.

In certain technical problems flows are encountered in channels which are formed as a result of the delivery of a fluid (or gas) through the channel walls (gas motion in a channel with porous walls, gas motion in the internal channels of powder charges, and fluid motion in wells). Laminar flows of this type were studied theoretically in [1] and experimentally in [2]. During the experimental study of the transition from the laminar flow regime to the turbulent regime in pipes and annular channels with porous walls in [3] it was discovered that the flow conditions along the front closed end (the channel bottom), where the results of the work [1] are not applicable, significantly affect the transition. /178*

The aim of the present work is determining the nature of flow in the initial segment of a flat channel both for a case of an impermeable bottom as well as for a case of delivery or withdrawal of fluid through the bottom. The investigation was accomplished by means of the numerical solution of Navier-Stokes equations by the finite-difference method.

1. We shall examine the stationary flow of a viscous incompressible fluid in a semi-infinite flat channel with parallel permeable walls, through which fluid is delivered at constant velocity v_0. The initial section of the channel is the y-axis and the x-axis and is directed along the channel axis. If for characteristic velocity and length we take indraft velocity v_0 and channel half-width a the flow under consideration in dimensionless variables will be described by the equation

$$\frac{1}{R_0}\Delta\Delta\psi-\frac{\partial\psi}{\partial y}\frac{\partial\Delta\psi}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial y}=0\quad\left(R_0=\frac{av_0}{\nu}\right). \tag{1.1}$$

*Numbers in margin indicate pagination in foreign text.

Here R_0 is the indraft Reynolds number, ν is the kinematic viscosity coefficient, ψ is the stream function, x and y are coordinates along and across the channel, and Δ is a Laplacian.

Since we are examining the flow in the initial segment of the channel we shall solve the problem in the region of $\Omega = \{0 \leqslant x \leqslant L,\ 0 \leqslant y \leqslant 1\}$, utilizing the flow symmetry relative to the channel axis. The boundary conditions for (1.1) will be the symmetry conditions on the channel axis

$$\frac{\partial\psi(x,0)}{\partial x} = \frac{\partial^2\psi(x,0)}{\partial y^2} = 0 \qquad (0 \leqslant x \leqslant L). \tag{1.2}$$

the indraft conditions and disappearing longitudinal velocity component on the permeable wall

$$\frac{\partial\psi(x,1)}{\partial x} = 1, \quad \frac{\partial\psi(x,1)}{\partial y} = 0 \qquad (0 \leqslant x \leqslant L) \tag{1.3}$$

and the adhesion conditions in the initial section

$$\frac{\partial\psi(0,y)}{\partial y} = \frac{\partial\psi(0,y)}{\partial y} = 0 \qquad (0 \leqslant y \leqslant 1). \tag{1.4}$$

The question of the boundary conditions on the right boundary (x = L) will be examined below. In the case of indraft or suction through the bottom of the channel the condition (1.4) is replaced by

$$\frac{\partial\psi(0,y)}{\partial x} = 0, \quad \frac{\partial\psi(0,y)}{\partial y} = b \quad \left(b = \frac{u_1}{v_0},\quad 0 \leqslant y \leqslant 1\right). \tag{1.5}$$

Here u_1 is fluid indraft (or suction) velocity through the bottom ($b > 0$ corresponds to indraft, while $b < 0$ corresponds to fluid suction).

2. In the case of symmetrical fluid flow with respect to the initial channel section, when in place of conditions involving adhesion to the bottom of the channel (1.4) conditions of symmetry occur

$$\frac{\partial\psi(0,y)}{\partial y} = \frac{\partial^2\psi(0,y)}{\partial x^2} = 0 \qquad (0 \leqslant y \leqslant 1) \tag{2.1}$$

and a solution to the problem exists, as obtained in [1] and has the form

$$\psi = xF(y). \tag{2.2}$$

For the function F(y) we obtain the equation /179

$$F'F'' - FF''' = \frac{1}{R_0} F'''', \tag{2.3}$$

which must be solved with boundary conditions

$$F(0) = F''(0) = F'(1) = 0, \quad F(1) = 1. \tag{2.4}$$

With large and small R_0 numbers the solution of the problem (2.3), (2.4) may be approximated in analytical form while in a general case a numerical integration of this boundary value problem is required. If for characteristic velocity we do not take v_0, but rather the velocity on the channel axis in given section u(x, 0) it follows from (2.2) that in all channel sections the velocities will be similar

$$\frac{u(x, y)}{u(x, 0)} = \frac{F'(y)}{F'(0)}. \tag{2.5}$$

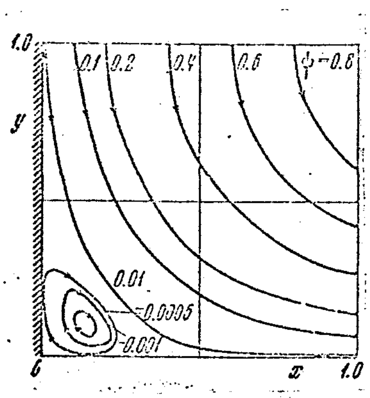

Figure 1.

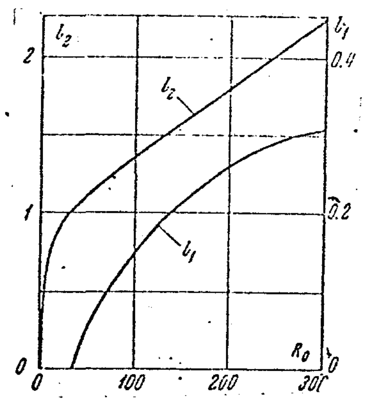

Figure 2.

Experimental results in the works [2] and [3] reveal that although conditions (1.4) or (1.5) are realized in experiments, rather than those in (2.1) velocity profiles (2.5) are established at a considerable distance from the bottom of the channel. Therefore we shall assume that in a case of fluid adhesion to the channel bottom at a considerable distance from it, the flow will be described by the relationships (2.2) and (2.3), which we shall use in order to obtain boundary conditions where x = L. For this we multiply (2.3) by x, and using (2.2) we obtain

$$\frac{\partial\psi}{\partial y}\frac{\partial^3\psi}{\partial x\,\partial y^2}-\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial y^3}=\frac{1}{R_0}\frac{\partial^4\psi}{\partial y^4}\,. \tag{2.6}$$

Similar boundary conditions are employed in [4-6].

3. The equation (1.1) with boundary conditions (1.2)-(1.4) and (2.6) was solved by the net-point method. For this purpose (1.1) was recorded in the form of a system relative to the stream function ψ and vorticity ω, which was then solved by the method established in [5]

$$\frac{\partial\omega}{\partial t}=\frac{1}{R_0}\Delta\omega-\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y} \tag{3.1}$$

$$\frac{\partial\psi}{\partial t}=\Delta\psi+\omega. \tag{3.2}$$

The difference plan for systems (3.1), (3.2) were written just as in [5]. The basic calculations were accomplished on a uniform net h = l = 0.05 (h and l are steps in the net in accordance with the x and y coordinates). The time step τ was chosen from the stability condition of the plan depending on the R_0 number. The calculation time for one variant according to the program compiled in the "ALGOL" language, where L = 2 (800 nodal points) on the M-20 ETsVM [electronic digital computer] amounted to 20-50 minutes, depending on the R_0 number.

4. We shall examine the boundary conditions for functions ψ and ω in connection with the solution of system (3.1), (3.2). We shall assume that $\psi(0.1)$ = 0. Then from (1.2)-(1.4) it is easy to obtain conditions

$$\begin{gathered}\psi(0,\,y)=0\qquad(0\leqslant y\leqslant 1)\\ \psi(x,\,0)=\omega(x,\,0)=0,\quad \psi(x,\,1)=x\qquad(0\leqslant x\leqslant L).\end{gathered} \tag{4.1}$$

The boundary conditions on the walls for ω were established in the form of dependencies on values ψ and ω at the limiting points with the aid of various approximations cited in the work [7]. The results of the calculations reveal that for the difference plan utilized the convergence velocity shows practically no dependence on the form of the boundary conditions employed. The condition (2.6) on the boundary x = L is written in the form /180

$$\frac{\partial\omega}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y}=\frac{1}{R_0}\frac{\partial^2\omega}{\partial y^2},\qquad \frac{\partial\psi}{\partial t}=\frac{\partial^2\psi}{\partial y^2}+\omega. \tag{4.2}$$

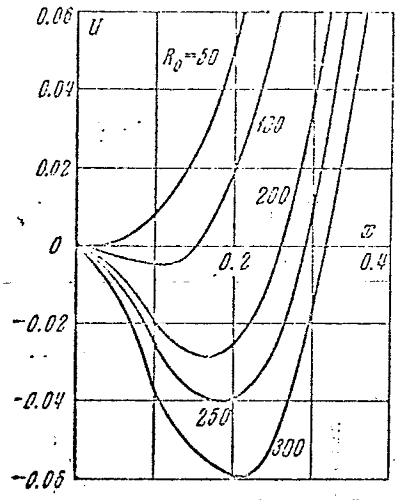

Figure 3.

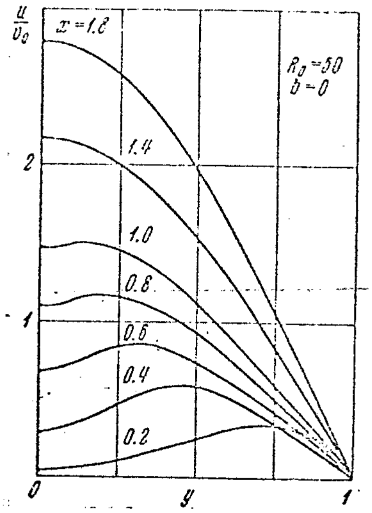

Figure 4.

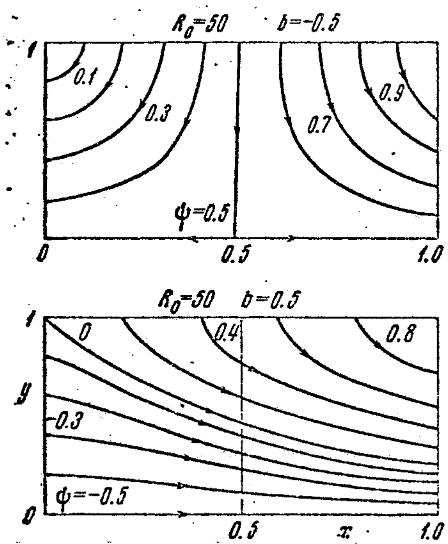

Figure 5.

Equations (4.2) were iterated simultaneously with the basic system (3.1), (3.2). Two different numerical applications (4.2) were employed. In the first of these expressions $\partial\omega/\partial x$ and $\partial\psi/\partial x$ from (2.2) were replaced by F" and F. In this case the values for ψ and ω on the boundary $x = L$ were determined independently of the nature of the flow where $x < L$ by the relationships (2.2) and (2.3) ("strict" boundary conditions). In the second application the expressions $\partial\omega/\partial x$ and $\partial\psi/\partial x$ were approximated on the $x = L$ with the aid of left linear three-point difference formulas, with an error of $O(h^2)$. Here the values ψ and ω on the boundary depend on the behavior of ψ and ω at limiting points ("mild" conditions). Numerical experiments have revealed that the application of strict conditions leads to boundary effects which significantly distort the solution at limiting points. Under less strict conditions such effects do not arise.

5. Numerical calculations were accomplished in order to determine the influence of the value R_0 on the nature of the flow in the initial segment of

channel where $10 \leqslant R_0 \leqslant 300$. In conjunction with the case of an impermeable channel bottom ($b = 0$) cases were examined for delivery ($b > 0$) or suction ($b < 0$) of fluid through the bottom.

Where $b = 0$ and with sufficiently large values R_0 a stagnation zone develops near the center of the bottom the form of a pair of symmetrical vortices, which slowly rotates in opposite directions. Figure 1 shows streamlines for $b = 0$ and $R_0 = 200$. The length of the rotational stagnation zone l_1 increases with increase in R_0 while at large values R_0 this increase becomes insignificant (Figure 2). Fluid velocity in the stagnation zone is very low and in the range of R_0 numbers under consideration the velocity does not exceed $0.06\ v_0$. Figure 3 gives velocity distribution $u(x, 0)/v_0$ along the axis near the channel bottom.

In order to evaluate the stability of flows in the base region the velocity distribution in the different sections is of interest. Figure 4 shows the profiles of longitudinal velocity components for $b = 0$, $R_0 = 50$ in different sections of the channel. In the initial sections the velocity profiles have points of inflection which, from the point of view of the theory of stability of parallel flows, indicates their extreme instability. At a certain distance from the channel bottom the velocity profiles take the form (2.5), obtained in [1] for symmetrical flow with respect to the initial section. We shall designate by l_2 the segment length on which the velocity profiles display points of inflection. The dependence l_2 on the R_0 is shown in Figure 2. At high values R_0 the value l_2 is proportional to R_0. /181

Experiments [3] revealed the intensive influence of flow conditions in the base region on the transition from laminar flow to turbulent flow in porous pipes. In particular it was found that the indraft to the bottom led to a decrease in the length of the laminar segment in the pipe and that suction led to its increase. Calculations made revealed for a fixed R_0 number with an increase in the indraft value, the value l_2 increased, while during suction it decreased. This is one of the possible explanations for the experimental facts cited above.

Figure 5 shows streamlines for $R_0 = 50$ in the case of indraft ($b = 0.5$) and suction ($b = -0.5$) through the bottom. In both cases the stagnation zones which occur for $b = 0$ do not arise. We note that in the case of comparatively high suction the streamlines have the same form as during symmetrical fluid flow. This means that in experiments it is possible to exclude the influence of the channel bottom and to investigate flow stability (2.2) while drawing off a part of the fluid through the bottom and computing the x coordinate from the flow point, rather than from the channel bottom.

REFERENCES

1. Berman, A. S., "Laminar Flow in Channels with Porous Walls", *J. Appl. Phys.*, Vol. 24, No. 9, p. 1232, 1953.
2. Taylor, G., "Fluid Flow in Regions Bounded by Porous Surfaces", *Proc. Roy. Soc. London*, Vol. A234, No. 1199, 1956.
3. Braylovskaya, I. Yu., "Calculation of Corner Flow by a Stream of Viscous Compressed Gas", *Izv. AN SSSR, MZhG.*, No. 3, 1967.
4. Varapayev, V. N., "Numerical Investigation of a Periodic Viscous Incompressible Fluid Jet Stream", *Izv. AN SSSR, MZhG.*, No. 3, 1968.
5. Varzhanskaya, T. S., "One Method of Presenting Boundary Conditions for Viscous Fluid Flow Problems", *Izv. AN SSSR, MZhG.*, No. 3, 1969.
6. Kuskova, T. V. and L. A. Chudov, "Concerning Approximate Vortex Boundary Conditions in the Calculation of Viscous Incompressible Fluid Flow", *Vychislitelnyye Metody i Programmirovaniye* [Calculation Methods and Programming], M.G.U. Press, No. 11, 1968.

Translated for the National Aeronautics and Space Administration under Contract No. NASw-1695 by Techtran Corporation, P.O. Box 729, Glen Burnie, Md. 21061